MATHEMATICS
for
SCIENTISTS

An Introduction

Michael Bulmer

2016

Contents

1 Numbers 1

Natural numbers 1

Representation of natural numbers 1

Operations on natural numbers and the laws of arithmetic 2

Prime numbers 3

The unique factorization theorem 3

The number of primes 5

The sieve of Eratosthenes 5

Extensions of the number system 5

Negative integers 6

Rational numbers 7

Irrational numbers 8

Exponentiation 10

Complex numbers 11

2 Geometry and Trigonometry 13

Euclidean geometry 13

Parallel and perpendicular lines 13

Triangles 16

Trigonometry 19

Right-angled triangle definition of trigonometric functions 19

Circular definition of trigonometric functions 22

Applications of trigonometry 24

The calculation of π 27

Analytic geometry 29

Transformations 29

Straight lines 31

The circle and the ellipse 32

The parabola 34

The hyperbola 38

Spherical geometry and trigonometry 39

3 Sequences, Series, Functions and Limits 43

Sequences and series 43

Arithmetic sequences and series 43

Geometric sequences and series 44

Euler's number *e* 45

The Fibonnaci sequence 47

Functions of real variables 48

Limits and continuity 48

Inverse functions 51

Polynomial functions 53

Exponential and logarithmic functions 54

4 Calculus 58

Differential Calculus 58

Differentiation 60

Exploring curves 63

Series expansions 66

Integral Calculus 69

The Riemann-Darboux integral 69

The fundamental theorem of calculus 71

5 Differential and Difference Equations 74

Differential Equations 74

Exponential decay 74

Logistic growth 77

Harmonic motion 80

Simple harmonic motion 80

Damped harmonic motion 81

Forced oscillations and resonance 84

Difference Equations 86

The Ricker model 86

6 Vectors and Matrices 89

Addition and Multiplication 89

Determinants and Matrix Inversion 91

Regression analysis 94
Eigenvalues and Eigenvectors 97
The Leslie matrix 99
Prey-predator models 100
Euclidean Vectors 104
Two-dimensional Euclidean vectors 104
Three-dimensional Euclidean vectors 107
One-dimensional Euclidean vectors 110

7 Newton's Laws of Motion 111
Newton's Laws 112
Apparent Forces 113
Gravity 118
Newton's shell theorem 119
Terrestrial gravity 121
Motion of a projectile 123
Motion of a pendulum 124
Kepler's Laws of Planetary Motion 125
Conic sections in polar coordinates 126
Kepler's second law 127
Kepler's first law 128
Kepler's third law 129

8 Tides 131
Tide-generating Force 133
Equilibrium Theory of the Tides 136
Development of the theory 137
Motion of the moon and sun 139
Spring and neap tides; lagging and priming 140
Effect of distance from the earth 143
Effect of declination 144

9 Waves 147
Vibrating Strings 147
The wave equation 148
d'Alembert's solution of the wave equation 149

Fourier analysis 152
Sound Waves 156
Water Waves 159
Sinusoidal water waves 160
Gravity waves 164
Wind waves, tsunamis, seiches and tidal waves 167

10 Probability Theory 174
The Two Laws of Probability 175
Mendelian genetics 177
Probability Distributions and Random Variables 181
Random variables 179
The binomial, Poisson and exponential distributions 182
The normal distribution 187
The multivariate normal distribution 189
The χ^2 *distribution* 192

11 Statistical Inference 194
Bayesian Inference 194
Classical Statistical Inference 196
Point estimation 197
Student's t test 200
The χ^2 *test of goodness of fit* 203
Regression 205
Analysis of variance 208

Exercises 211

Answers 219

The Greek Alphabet 227

Further Reading 228

Index 229